Nonsmooth and Set-Valued Analysis in the V.V.Goncharov's vision

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Introduction

In this short presentation, I will try to summarize the aims and the vision of Vladimir Goncharov towards nonsmooth and set-valued analysis, as I was able to understand from his book.

As he explained me during one of his visits to the University of Verona, his aim was not to write a huge treatise to please a very restricted public of experts, but rather to popularize nonsmooth and set-valued analysis for a broader public, for instance of students and young researchers, showing in action the powerful tools of these two disciplines, in particular in the field of dynamic optimization, but at the same time without neglecting a rigorous presentation of concepts and constructions.

I believe that everybody can clearly seen this attitude throughout the pages of his opera, that also tried to made as self-contained as possible.

The mathematics of optimization

According to Goncharov's point of view, many features of the behavior of a (differentiable) map can be reconstructed by studying the set of tangent lines to the points of the graph. Unfortunately, as can be shown by simple examples at the beginning of the first part of the book, in the study of optimization problems, it often occurs that the involved functions fail to be classically everywhere differentiable.

Perhaps the most simpler but also the most important example of this behavior is provided by the distance function from a given closed set, which in general may fail to be differentiable unless further assumptions are added.

The fact that optimization problems are in general intrinsecally nonsmooth motivates the need of constructions able to generalize the classical derivatives.

From convex to nonsmooth analysis

As a first class of possibly nondifferentiable functions involved in many important optimization problems, Goncharov introduces the class of convex functions. It is important to stress how many concept, like e.g. the subdifferential of convex analysis, before being introduced in a rigorous mathematical way, are addressed first in a informal geometrical way, in order to allow the reader to visualize the new concepts as natural generalizations of previous ones holding in a smooth framework.

The constructions made for convex analysis are extended to nonconvex function in the second chapter. Also in this case it is stressed the continuity between the convex objects and their nonconvex generalizations.

Why we need set-valued analysis?

The second part of the book is devoted to set-valued analysis, i.e., the study of the properties of the correspondences that associates to each value of the independent variable a subset of the target space.

As in the first part, the motivations for the study of such a class of maps are provided by many examples ranging from economical interpretation of some examples in game theory, to the problem of the invertibility of single-valued maps, or to the need to generalize the theory of ordinary differential equations to the theory of differential inclusions, which provides a nice *geometrical* interpretation of many control systems.

To enhance again the continuity with respect to the previous part, it is addressed the study of the subdifferentials as set-valued maps.

Nihil recte sine exemplo docetur aut discitur

It must be stressed the high number of exercises and examples disseminated in all the chapter, and the section devoted to applications of the explained theory, as well as the many advices and remarks on the statements and the proofs.

In this way the reader feels to play an active role in the learning process, not just passively contemplating a construction made by others, but rather building up it step by step, with a clear motivation even for the more abstract and difficult parts.

Sparse Conclusions

Throughout the pages of the book we can really see the commitment of Vladimir Goncharov to illustrate concepts in the simplest way as possible, but without sacrificing rigor and a solid mathematical foundation.

In the same years in which he started to write this book, he was used to give a short course on Nonsmooth and Set-Valued Analysis at the University of Verona, constantly improving it on the bases of the feedback received by our students, in order to better transmit - besides the mathematical theory - his curiosity and passion toward this kind of problems.

I think that this is really an important part of his legacy.