ENCONTRO DE MATEMÁTICA EM MEMÓRIA DE VLADIMIR GONCHAROV

LIVRO DE RESUMOS

DEPARTAMENTO DE MATEMÁTICA

ESCOLA DE CIÊNCIAS E TECNOLOGIA, UNIVERSIDADE DE ÉVORA

30 DE NOVEMBRO DE 2018

Este encontro tem como objetivo homenagear a memória do Professor Doutor Vladimir Goncharov, natural de Irkutsk, Rússia, que tragicamente nos deixou em novembro de 2017. O seu desaparecimento representa uma enorme perda para o Departamento de Matemática da Escola de Ciências e Tecnologia da Universidade de Évora, onde lecionava, e para a investigação em Matemática, que fazia com o maior prazer. Tinha interesse em várias áreas da Matemática, nomeadamente em Análise Multívoca, Análise não-suave, Análise Convexa, Inclusões Diferenciais, Cálculo das Variações, Controlo Óptimo e Processos Estocásticos.

No início do encontro será feito o lançamento do livro que o Vladimir estava a preparar para as suas provas de agregação, e no qual trabalhou até às últimas horas da sua vida.

Comissão Organizadora

Clara Carlota (Universidade de Évora)

Fátima Pereira (Universidade de Évora)

Luís Bandeira (Universidade de Évora)

Sandra Vinagre (Universidade de Évora)

Vladimir Bushenkov (Universidade de Évora)

Projeto UID/MAT/04674/2013

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Resumos

Some aspects of differential inclusions in Wasserstein space and applications

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In this talk we will present some recent results on differential inclusions theory in the space of Wasserstein in the framework of mean field control problems. The main motivation for this study is given by the control of multi-agent systems, where the number of the agents is so large that only a statistical description of the system is actually possible. These problems can be naturally casted as control problems in the space of probability measures, governed by a suitable lifted dynamics in the Wasserstein space.

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Lipschitz regularity for minimizers based on a new necessary condition for nonautonomous Lagrangians that are highly discontinuous in the state and velocity

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This is a joint work with Piernicola Bettiol.

We consider minimizer of the classical problem of the calculus of variations

$$\begin{cases} \text{Minimize} & I(x) := \int_a^b \Lambda(t, x(t), x'(t))\, dt \\ \text{subject to:} & x \in W^{1,1}([a,b];\mathbb{R}), x(a) = A, x(b) = B. \end{cases} \tag{P}$$

where $\Lambda : [a,b] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is just Borel measurable. It is widely known that if L is autonomous and superlinear, then any minimizer of (P) is Lipschitz. When L is non autonomous, some famous counterexamples show that this in general is not true anymore. At the same time no other regularity result for nonautonomous Lagrangians gives back, when there is not dependence on t, the most general result in the autonomous case. We formulate [2] a local Lipschitz assumption on $t \mapsto L(t, x, \xi)$, that extends to the nonautonomous case not only the most general versions of Lipschitz regularity theorem for autonomous and superlinear Lagrangians [4], but also the results obtained in the same framework under a Cellina's growth condition [3, 5]. This is obtained via what we consider a new necessary condition (W), established in [1], which shows a hidden convexity regularity. The proof of (W) relies on the more recent versions of Clarkes's Maximum Principle. As a byproduct we obtain a convex version of the DuBois-Reymond equation, in a nonsmooth setting and...without convexity assumptions of any kind. The results can be extended to local minima, in the senso of the $W^{1,1}$ norm, and we can cover essentially in (P) any kind of given constraints on the trajectories or on their derivatives.

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Sublinear-type conditions for second order impulsive coupled systems

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This work considers a second order impulsive coupled system with full nonlinearities, generalized impulse functions and mixed boundary conditions. This is the first time where such coupled systems are considered with nonlinearities with dependence on both unknown functions and their derivatives, together impulsive functions given by more general framework allowing jumps on the both functions and both derivatives

The arguments apply the fixed point theory, Green's functions technique, L^1-Carathéodory functions theory and Schauder's fixed point theorem.

An application to the transverse vibration system of elastically coupled double-string will be presented.

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Metric projections in Hilbert spaces and Moreau's sweeping process: how Volodya's ideas lead to new developments in those topics

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In 1998 Volodya and me started working together on a nonconvex version of Moreau's sweeping process, as usual starting from examples. It became clear to us that uniqueness (in finite dimensional spaces) and existence (in general Hilbert spaces) depend on curvature properties of the moving set. A little later, it became clear that everything was connected with existence and uniqueness of the metric projection on not necessarily convex sets. We identified a class of sets with good properties and Volodya proved a nice existence and uniqueness result for the metric projection. This was a really nice result. In fact, so nice that it was already proved at least by two other groups of people, completely independent one from the other. However, this result inspired most of my research work and lead to new developments, including continuous selections from a class of non convex valued set-valued maps.

In the class of sets that we (together with other people, independently) identified, it is possible to prove also existence and uniqueness of solutions of Moreau's sweeping process and also to derive necessary conditions for a class of optimal control problems involving such a dynamics.

Variational Problems of Plastic Surgery

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Surgery simulation is a growing field of biomechanics. It involves different areas of knowledge from anatomy to mathematics, to image processing. The number of studies in this area has significantly increased during the last decades. Usually surgery simulators use mass-spring models of soft tissues or finite element models based on differential static equilibrium equations from elasticity theory. However, the use of direct methods of Calculus of Variations allows one to apply powerful methods of numerical optimization in order to effectively solve the problem. In this talk we deal with so called general variational problem of plastic surgery. This is a problem of calculus of variations with unusual boundary conditions, known as knitting or suturing conditions. We discuss the existence of solutions to this problem, some computational aspects, and present illustrative examples.

Stability loss in singularly perturbed differential inclusions

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We study the differential inclusion

$$\varepsilon \frac{dx}{dt} \in tx + C, \tag{1}$$

for small ε and C an exponentially small convex set, which is symmetric with respect to 0. For $C = \{0\}$, we have the ordinary singular perturbation

$$\varepsilon \frac{dx}{dt} = tx. \tag{2}$$

It is the simplest equation with a *canard* solution, this means a solution which is stable until becoming close to the singular point $(0,0)$, then it will loose stability, but it will not go off to infinity, but instead remains close to the *slow curve* $x = 0$ during a non-neglectable period. Canards were discovered earlier as cycles in the Van der Pol equation [1], and it was Callot [2] who came with the simple local model (2). Callot also showed that a change in the equation (2), which is more than exponentially small with respect to ε, destroys the canard phenomenon. Nowadays canard solutions are detected in various biological models (e.g. [3]), and among others describe predator-prey coexistence and neurological spikes.

Lobry suggested me to study (1) with C a *neutrix*, meaning a nonstandard convex subgroup of the nonstandard real numbers; by Callot's result we know that the neutrix must be exponentially small. It happened to be possible to give an explicit solution, which comes as a set-valued function. An interesting property is the fact that we can almost predict at which time the solution leaves the slow curve, but not whether it leaves upward or downward: this is determined by the values of sections close to the singular point.

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Sweeping processes and evolution problems

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Sweeping processes are special evolution problems, where a crucial role is played by the normal cones to moving sets. Introduced by Jean Jacques Moreau in the early seventies, with applications in Mechanics in mind (quasistatics, dynamics), it has grown in applications, such as crowd motion, hysteresis, plasticity and many others. Vladimir Goncharov and Giovanni Colombo (1999) gave important contributions to the generalisation of the existence results to classes of nonconvex sets, such as φ-convex or prox-regular sets, and to the study of properties of such sets (2001). The talk aims at presenting briefly a few recent (or not so recent) generalisations to further classes of nonconvex sets and to evolution problems, governed by maximal montone operators with variable domains.

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On the total ith-mean curvatures of hypersurfaces

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The total mean curvature of degree i of a hypersurface $N \subset M$ is, essentially, the integral of the ith-degree symmetric polynomial on the principal curvatures of N. The principal curvatures are the eigenvalues of the second fundamental form of the hypersurface isometrically embedded in a Riemannian manifold (M, g). In this talk we consider M to be of constant sectional curvature. We quickly review a new proof of the Hsiung-Minkowski identities and then try to show that a certain *new* integral quantity is a constant of the hypersurface, under C^1 deformations. In other words, our result implies that we must be in the presence of the Theorem of Chern-Gauss-Bonnet: the integral constant is a topological invariant of N. For the whole purpose, we recur to a fundamental exterior differential system of Riemannian geometry. Such is a main subject of research followed in recent years.

Acknowledgements: This talk has been partially supported by Centro de Investigação em Mateática e Aplicações (CIMA), through the grant UID-MAT-04674-2013 of FCT-Fundação para a Ciência e a Tecnologia, Portugal.

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